# A Refinement of the Atomic and Thermal Parameters of $\alpha$-Manganese from a Single Crystal* 

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#### Abstract

The growth of small single crystals of $\alpha$-manganese has made possible the confirmation of the structure of this element determined previously by powder methods. This X-ray study has led to a precise structure determination and values for the Debye temperatures of the different atom types. $\alpha$-Manganese, the low-temperature allotrope, has a body-centered structure with 58 atoms per cell occupying four different symmetry sites in space group $I \overline{43 m}\left(T_{d}^{3}\right)$. The lattice constant of $8.911 \pm 0.002 \AA$ leads to a calculated density of $7.463 \mathrm{~g} . \mathrm{cm}^{-3}$. Integrated intensities for 1142 reflections ( 650 independent) were measured by counter methods using Mo $K \alpha$ radiation. The data were refined by the least-squares method to yield a conventional $R$ factor on $F^{2}$ of $8.4 \%$.


## Introduction

The structure of $\alpha$-manganese has been studied previously by several workers. Westgren \& Phragmén (1925), in an X-ray investigation of the allotropy of manganese, were able to determine the lattice parameter. Bradley \& Thewlis (1927), using intensities from Westgren \& Phragmén's photographic data of $\alpha$-manganese powder, correctly determined the space group ( $I \overline{4} 3 \mathrm{~m}$ ) and the number of atoms per unit cell (58). Values for the fractional coordinates of the atoms were also determined. These results were confirmed (Preston, 1928) from oscillating crystal photographs of nearly single-crystal samples. Gazzara, Middleton, Weiss \& Hall (1967) reported a refinement of these atomic positions based on several sets of X-ray intensities obtained from $\alpha$-manganese powder using Cr , Mn , and Cu radiation and counter methods. Atomic positions have been refined from single-crystal X-ray data by Kunitomi, Yamada, Nakai \& Fujii (1969). $\ddagger$ The results of these investigations are given in Table 1.
$\alpha$-Manganese is unusual among metals and elements in the complexity of its crystal structure. The variations in interatomic distance and coordination number among the four atom types makes the structure of $\alpha$-manganese more like that of an intermetallic compound than an element. The growth of pure single crystals of $\alpha$-manganese in this laboratory has made possible a more precise determination of the structure

[^0]of this element. The present X-ray study was undertaken for several reasons. First, it is of intrinsic importance to know the structures of the elements as accurately as possible. Second, antiferromagnetism has been observed in $\alpha$-manganese below $95^{\circ} \mathrm{K}$ (Shull \& Wilkinson, 1953; Kasper \& Roberts, 1956). This present X-ray study was desirable as a prelude to a neutron diffraction study (Oberteuffer, Marcus, Schwartz and Felcher, 1968, 1970; Oberteuffer, 1969) of the magnetic structure. Third, the determination of the individual characteristic temperatures of the four atom types is possible from single-crystal data and is of importance since surprisingly large differences in these temperatures were hypothesized to explain the Mössbauer spectra of ${ }^{57} \mathrm{Fe}$ in an alloy of $5 \% \mathrm{Fe}$ in $\alpha$-manganese (Kimball, Phillips, Nevitt \& Preston, 1966).

The sample used in this experiment was prepared from a crystal grown by a vapor deposition method (Oberteuffer, Marcus, Schwartz \& Felcher, 1970). To minimize absorption effects the crystals were ground into spheres with radii of the order of 0.2 mm . They were then etched in solutions of $10 \% \mathrm{HCl}$ in methanol to reduce the radius by about half. It was difficult to prevent some asphericity from developing during etching even though the etching solution was agitated. The smallest sphere that could be prepared in this way was about $0 \cdot 1 \mathrm{~mm}$ in radius. The sample used in this experiment was an ellipsoid whose principal axes were $0.11,0.084$ and 0.068 mm . This crystal was mounted with the long axis of the ellipsoidal crystal roughly parallel to the $\varphi$ axis of a Picker four-circle automatic diffractometer.

## Collection and reduction of intensity data

Fourteen reflections were manually centered on the diffractometer and their setting angles formed the basis
for a least-squares refinement* of cell constant and orientation as previously described (Corfield, Doedens \& Ibers, 1967). The lattice constant derived was $8 \cdot 911(2)$ $\AA$, based on observations made at $20^{\circ} \mathrm{C}$ with Mo $K \alpha_{1}$ radiation ( $\lambda=0.70930 \AA$ ).

Intensity data were collected in the manner previously described (Corfield, Doedens \& Ibers, 1967). The diffracted beams were filtered through a 3 mil niobium foil. The $\theta-2 \theta$ scan technique was used, at a scan rate of $1^{\circ}$ per minute, with scans ranging from $0.5^{\circ}$ in $2 \theta$ below the $K \alpha_{1}$ peak to $0.5^{\circ}$ above the $K \alpha_{2}$ peak. Stationary-counter, stationary-crystal background counts at the extreme positions were made for 20 seconds each. The pulse height analyzer was set to accept a $90 \%$ window. The diffracted beams passed through a counter aperture of dimensions $7 \times 7 \mathrm{~mm}$ which was positioned 29 cm from the crystal. At a take-off angle of $1 \cdot 1^{\circ}$, where a typical peak height was $80 \%$ of the maximum as a function of take-off angle, the widths at half-height of the scans were only about $0 \cdot 1^{\circ}$ in $2 \theta$. Thus the eventual problems with extinction were not surprising.
During the data collection three reflections were chosen as standards and were observed every 200 re-

[^1]flections. No differences were observed other than those expected from the usual statistical deviations. A total of 1142 reflections was collected out to $\lambda^{-1} \sin \theta$ of $1 \cdot 3 \AA^{-1}$ (using both the bisecting and parallel geometries). Of these, 129 were less than their standard deviations. This data set consisted of two members of the $h k l$ form, specifically $h k l$ and $h l k$.

The data were reduced as described previously, using a value for $p$ of $0 \cdot 03$. Absorption effects are significant even with an average sample radius of 0.085 mm since $\mu$ is $257 \mathrm{~cm}^{-1}$ for $\alpha$-manganese using Mo $K \alpha$ radiation. The attenuation factor of the radiation over the mean radius is approximately 10 . Initially a spherical absorption correction was applied to the data using the mean radius of 0.085 mm for the ellipsoidal crystal. In the final stages of refinement a more realistic absorption correction was applied: The ellipsoidal crystal was approximated by a 30 -sided polygon and a numerical absorption correction was made. The transmission factors calculated in this way varied from about 0.06 at low angles to 0.14 at high angles. The anisotropy of the correction because of the non-spherical shape was as large as $50 \%$ at low angles and decreased to $20 \%$ at high angles. The maximum error in the local radius of the crystal as a result of the polygonal approximation is of the order of $6 \%$, but microscopic examination of the crystal revealed surface irregularities of at least this magnitude. Because of the large absorption much of the intensity of a reflection comes from the surface of the crystal and these surface ir-

Table 1. Atomic positional parameters

|  | Atom I | Atom II | Atom III | Atom IV | Lattice parameter |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Wyckoff notation | 2 (a) | $8(c)$ | 24(g) | 24(g) |  |
| Site symmetry | 43 m | $3 m$ | $m$ | $m$ |  |
| Equivalent positions | 000 | $x x x$ | $x x z$ | $x x z$ | $a$ |
| Bradley \& Thewlis | 000 | $x=0.317$ | $\begin{aligned} x & =0.356 \\ z & =0.042 \end{aligned}$ | $\begin{aligned} & x=0.089 \\ & z=0.278 \end{aligned}$ | $8.894 \AA$ |
| Gazzara, Middleton, Weiss \& Hall | 000 | $x=0.316$ | $\begin{aligned} x & =0.356 \\ z & =0.034 \end{aligned}$ | $\begin{aligned} x & =0.089 \\ z & =0.282 \end{aligned}$ | 8.9125 (10)* |
| Kunitomi, Yamada, Nakai \& Fujii | 000 | $x=0.3176$ | $\begin{aligned} x & =0.3569 \\ z & =0.0346 \end{aligned}$ | $\begin{aligned} x & =0.0898 \\ z & =0.2820 \end{aligned}$ |  |
| Present experiment | 000 | $x=0.31787$ (10) | $\begin{aligned} & x=0.35706 \\ & z=0.03457 \end{aligned}$ | $\begin{aligned} & x=0.08958 \\ & z=0.28194 \end{aligned}$ | $8 \cdot 911$ (2) |

Table 2. Results of various refinements

| Refinement number | $h k l, h l k$ averaged | Absorption correction | Debye-Waller factors | Scattering factor | $R_{w}$ | $R_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , | no* | Spherical | Isotropic | Mn 0 | 16.0\% | 14.2\% |
| 2 | no* | Ellipsoidal | Isotropic | Mn 0 | $13 \cdot 3$ | 11.8 |
| 3 | no* | Ellipsoidal | Anisotropic | Mn 0 | $13 \cdot 2$ | 11.7 |
| 4 | yes $\dagger$ | Ellipsoidal | Isotropic | Mn 0 | $12 \cdot 4$ | $9 \cdot 7$ |
| 5 | yes | Ellipsoidal | Isotropic | $\mathrm{Mn}^{2+}$ | $11 \cdot 3$ | $9 \cdot 2$ |
| $6 \ddagger$ | yes | Ellipsoidal | Isotropic | $\mathrm{Mn}^{2+}$ | $11 \cdot 5$ | $8 \cdot 4$ |

* Number of observations $=1142$.
$\dagger$ Number of observations $=650$.
$\ddagger \sigma\left(F^{2}\right)$ adjusted to give an error in an observation of unit weight the value $1 \cdot 0$ for all classes of reflections, as a function of $F_{0}{ }^{2}$, and as a function of $\theta$.
regularities are thus a significant contributor to the uncertainty of the low angle data. At low angles the $R$-factor for averaging equivalent reflections was $8.9 \%$ before the final absorption correction and $8.5 \%$ after. For all reflections, however, averaging $h k l$ and $h l k$ gave $R$ factors of $7.9 \%$ before and $6.1 \%$ after the absorption correction. Thus there is some justification for the use of the more sophisticated absorption correction.


## Refinement of the structure

In the full-matrix least-squares refinements, the function minimized was $\sum w\left(F_{o}^{2}-F_{c}^{2}\right)^{2}$, where $w$, the weight, is taken as $1 / \sigma^{2}\left(F_{o}^{2}\right)$ and where $F_{o}^{2}$ and $F_{c}^{2}$ are the squares of the observed and calculated structure amplitudes. The conventional and weighted $R$ factors are defined as $R_{1}=\sum\left|F_{o}^{2}-F_{c}^{2}\right| / \sum F_{o}^{2}$ and $R_{w}^{2}=\sum w\left(F_{o}^{2}-F_{c}^{2}\right)^{2} /$ $\sum w F_{o}^{4}$. The $\mathrm{Mn}^{0}$ (and later $\mathrm{Mn}^{2+}$ ) scattering factors were taken from Watson \& Freeman (1961) and Freeman \& Watson (1961). The values of $\Delta f^{\prime}$ and $\Delta f^{\prime \prime}$ were taken from the calculations of Cromer (1965) and were applied to the calculated structure factors (Ibers \& Hamilton, 1964).

The course of the refinement is shown in Table 2. Initially the parameters varied were the overall scale factor, the atomic coordinates of the atoms (starting with those values in Table 1), the isotropic DebyeWaller factors for the four independent atoms, and a one-parameter extinction parameter (Zachariasen, 1968). Space group $I \overline{4} 3 m$ was assumed.

The final values of $F_{o}^{2}$ and $F_{c}^{2}$ (in electrons ${ }^{2} \times 0 \cdot 1$ ) are given in Table 4. The final parameters are listed in Tables 1 and 3. The extinction coefficient in absolute units was $4.0(3) \times 10^{-5}$. Although the various refinements indicated in Table 2 led to slight differences in the derived Debye-Waller factors, they did not lead to significant differences in the positional parameters.

A final difference Fourier synthesis has no interpretable features. The maximum density is $2.9 \mathrm{e} . \AA^{-3}$, about $1 \%$ of the manganese peaks on an observed electron density map.

Table 3. Thermal parameters determined for $\alpha$-manganese

|  | Debye-Waller factor $B$ | Debye temperature $\Theta_{D}$ | Root-mean square displacement |
| :---: | :---: | :---: | :---: |
| I | $0.447(23) \AA^{2}$ | 379 (11) ${ }^{\circ} \mathrm{K}$ | 0.075 (2) A |
| II | $0 \cdot 454$ (12) | 376 (5) | 0.076 (1) |
| III | $0 \cdot 423$ (7) | 390 (4) | 0.0732 (7) |
| IV | $0 \cdot 391$ (7) | 406 (4) | 0.0704 (6) |
| Average | $0 \cdot 414$ | 394 | 0.0725 |

## The structure

The structure is that deduced from earlier powder data. A stereoscopic view of the unit cell is shown in Fig. 1. The type I atom at the body-centered position is surrounded tetrahedrally by four atoms of type II (these four shown are actually those belonging to the corners of the unit cell). At a somewhat larger distance twelve type III atoms form a polygon whose faces are triangles and rectangles. Closest to atom I are the twelve type IV atoms. Fig. 2 shows the four atom types and their nearest neighbors in stereo pairs. In Table 5 the interatomic distances for the nearest neighbors of the four atom types are listed.

The thermal parameters determined in this experiment are listed in Table 3. A refinement using anisotropic thermal ellipsoids did not improve the value of $R_{w}$ nor did it lead to values that differed significantly from the spherical model. The Debye temperature was calculated from the simple Debye model for cubic crystals with one atom per unit cell. In a previous determination of the average Debye temperature Gazzara, Middleton \& Weiss (1964) found $\Theta_{D}=390^{\circ} \mathrm{K}$ from X-ray reflection intensity ratios at different temperatures. Kimball, Phillips, Nevitt \& Preston (1966) obtained $\Theta_{D}=410^{\circ} \mathrm{K}$ for Fe atoms in sites III and IV in a $5 \% \mathrm{Fe}-\mathrm{Mn}$ alloy with the $\alpha$-manganese structure from the temperature variation of the total absorption of Mössbauer spectra. They suggested that atoms III and IV might have the widely different individual


Fig.1. Stereoscopic view of one half of the unit cell. The 28 nearest neighbors of the type $I$ atom at the body-centered position are shown. The unit-cell corners are defined by 8 type I atoms. Each corner and body-centered position in the lattice has this coordination. The atoms are differentiated by size, type I is the largest, type IV the smallest.

Debye temperatures of 500 and $250^{\circ} \mathrm{K}$. The absolute values of the Debye temperatures are sensitive to the magnitude of the absorption correction, which is not precisely known, and to the effects of thermal diffuse scattering, which were ignored in the refinement. But the relative values of $\Theta_{D}$ are insensitive to the absorp-
tion correction and are probably not very sensitive to the effects of thermal diffuse scattering. We conclude, therefore, that although some significant variations in the Debye temperatures of the atom types exist, these variations are far smaller than suggested by Kimball, et al.

Table 4. Values of $0 \cdot 1 F_{o}^{2}$ and $0 \cdot 1 F_{c}^{2}$ (in electrons ${ }^{2}$ ) for $\alpha$-manganese



Fig.2. Stereoscopic views of the coordination of the four atom types. These views are from the same angle as that of Fig.1. Bond lengths not explicitly indicated are the same as those given for atoms marked with similar symbols.

Table 5. Interatomic distances for nearest neighbors
$\begin{array}{ccccc}\begin{array}{c}\text { Atomic } \\ \text { type }\end{array} & \begin{array}{c}\text { Nearest neighbors } \\ \text { Num- } \\ \text { ber }\end{array} & \text { Type } & \begin{array}{c}\text { Distance* } \\ \text { I }\end{array} & 4\end{array}$ II $\left.\begin{array}{c}\text { Cordination } \\ \text { number }\end{array}\right\}$

One might expect differences to exist in the true scattering factors of the four different types of manganese atoms, since the coordination number and symmetry of their sites are different. Although such effects were looked for in a second data set which consisted of all dependent reflections in the region $\lambda^{-1} \sin \theta$ $<0 \cdot 3 \AA^{-1}$, no definite conclusions could be reached. This is not surprising since these inner data are most sensitive to the model for extinction and are affected more by the problems of the absorption correction.

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# The Crystal and Molecular Structure of $\alpha, \alpha^{\prime}$-Diselenobisformamidinium Dichloride 

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The crystal structure of $\alpha, \alpha^{\prime}$-diselenobisformamidinium dichloride, $\left[\mathrm{SeC}\left(\mathrm{NH}_{2}\right)_{2}\right]_{2} \mathrm{Cl}_{2}$, has been determined by a three-dimensional X-ray analysis ( $\mathrm{Cu} K \alpha$ ). The unit cell of dimensions $a=12 \cdot 82$ (2), $b=6 \cdot 01(1), c=12 \cdot 22(1) \AA, \beta=108 \cdot 1(0 \cdot 1)^{\circ}$ contains 4 stoichiometric units (space group $C 2 / c$ ). The organic cation is formed by two selenourea groups nearly perpendicular to the $\mathrm{Se}-\mathrm{Se}=2 \cdot 380$ (6) $\AA$ bond, the dihedral angle $\mathrm{CSeSe}{ }^{\mathrm{i}} \wedge \mathrm{SeSe}^{\mathrm{i}} \mathrm{C}^{\mathrm{i}}$ being $89 \cdot 5^{\circ}$; bond distances in those groups are: $\mathrm{Se}-\mathrm{C}=$ $1 \cdot 94$ (1), $\mathrm{C}-\mathrm{N}=1.32$ (2) and 1.30 (2) $\AA$. All the hydrogen atoms are involved in $\mathrm{N} \cdots \mathrm{Cl}$ hydrogen bonds. The molecular structure of the cation is compared with that of the corresponding thio-derivative.

## Introduction

In an attempt to prepare silver(I)-selenourea complexes, a few crystals of $\alpha, \alpha^{\prime}$-diselenobisformamidinium
dichloride were obtained. They were probably formed by oxidation of selenourea by the $\mathrm{Ag}^{+}$ion or by atmospheric oxygen. As the crystals were too few for a conventional chemical analysis, their nature was deter-


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    $\ddagger$ This work was reported in abbreviated form after submission of the present manuscript.

[^1]:    * In addition to various local programs for the CDC 6400 local versions of the following programs were used: ORFLS least-squares refinement and $O R F F E$ error function programs of Busing \& Levy, GONO absorption correction program of Hamilton, FORDAP Fourier program of Zalkin, and ORTEP plotting program of Johnson.

